Effect of natural convection on stability of flow in a vertical pipe

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If water is heated or cooled while flowing through a vertical pipe with a laminar motion, the velocity profile will differ from the parabolic shape for isothermal flow due to density variations in the fluid. If a constant heat flux is used at the wall and if the changes in temperature affect only the density appearing in the gravity term of the equations of motion, a condition is attained far downstream in the heat-transfer section such that there is no further change in the velocity profile. The shape of this fully developed velocity profile depends on the ratio of the heat flux to the flow rate. The stability of flow in an electrically heated pipe 762 diameters long was studied by detecting temperature fluctuations in the effluent. By use of a carefully designed entry and a long isothermal section prior to the heat exchange section, inlet disturbances were eliminated and transition to an unsteady flow resulted from a natural instability of the distorted profiles. It was found that the stability depends primarily on the shape of the velocity profile and only secondarily on the value of the Reynolds number, if at all. For upflow heating the flow first becomes unstable when the velocity profiles develop points of inflexion. Transition to an unsteady flow involves the gradual growth of small disturbances and therefore it is quite possible to have unstable flows without observing transition because the pipe is not long enough for the disturbances to attain a measurable amplitude. For downflow heating the flow instability is associated with separation at the wall. Transition to an unsteady flow is sudden and therefore transition occurs shortly after an unstable flow occurs. It is suggested that a change from a steady symmetrical to a steady unsymmetrical flow occurs in downflow when the profile develops points of inflexion.

Introduction

The velocity profile of a fluid flowing with a laminar motion through a heated or cooled section of pipe will differ from the parabolic profile that exists for fully developed isothermal flow due to variation of fluid density and viscosity with temperature over the cross-section of the pipe. For water, heating or cooling can cause transition to an unsteady flow at Reynolds numbers much lower than are usually associated with transition to turbulence for isothermal flow. This

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paper describes the results of experiments which were performed to relate this transition to distortions of the flow field caused by density variation.

Two types of heat-transfer experiments are conveniently conducted, one using a constant heat flux at the wall and the other using a constant-temperature wall. For the case of a constant heat flux, if the changes in temperature affect only the density appearing in the gravity term of the equations of motion, a condition is attained far downstream in the heat-transfer section such that there is no further change in the velocity profile and such that the temperature and pressure vary linearly with distance downstream. The maximum distortion of the

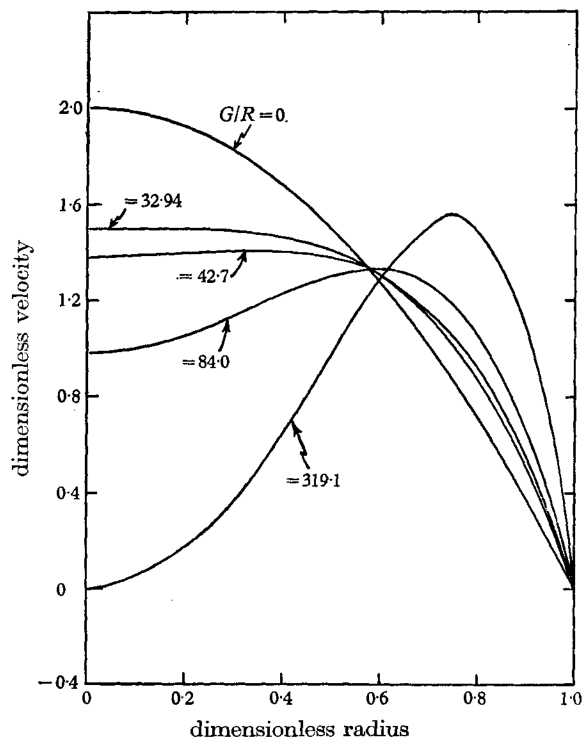

FIGURE 1. Velocity profiles for laminar upflow with constant-flux heating.

velocity profile for a completely laminar flow occurs in this region of fully developed flow. An analytical solution for the fully developed velocity fields, presented by Hanratty, Rosen & Kabel (1958), by Hallman (1956), and by Morton (1960) for symmetrical flow, is shown in figures 1 and 2. The degree of distortion of the velocity profile may be described as a function of the ratio of two dimensionless groups G/R. The group R is the Reynolds number using the radius of the pipe, a, as a length parameter. The group G, which is a measure of the rate of heat input per unit area, q, is defined as $a^4Bgq/k\nu^2$, where B, k, and ν are the coefficient of expansion, the thermal conductivity and the kinematic viscosity of the fluid, respectively, and g is the acceleration of gravity.

When natural convection is in the direction of forced flow (heating in upflow) the effect of density variation with temperature is to increase the flow velocity near the wall causing a flattening of the velocity profile in the centre of the pipe. At G/R greater than 32·94, the profiles have a dimple at the pipe centre and the point of maximum velocity moves radially to the tube wall as the ratio of heat flux to flow rate is increased. There is a reversal of flow at the centre of the pipe for G/R greater than 319·1. All profiles with G/R greater than 32·94 have points of inflexion. If natural convection is opposed to the direction of forced flow

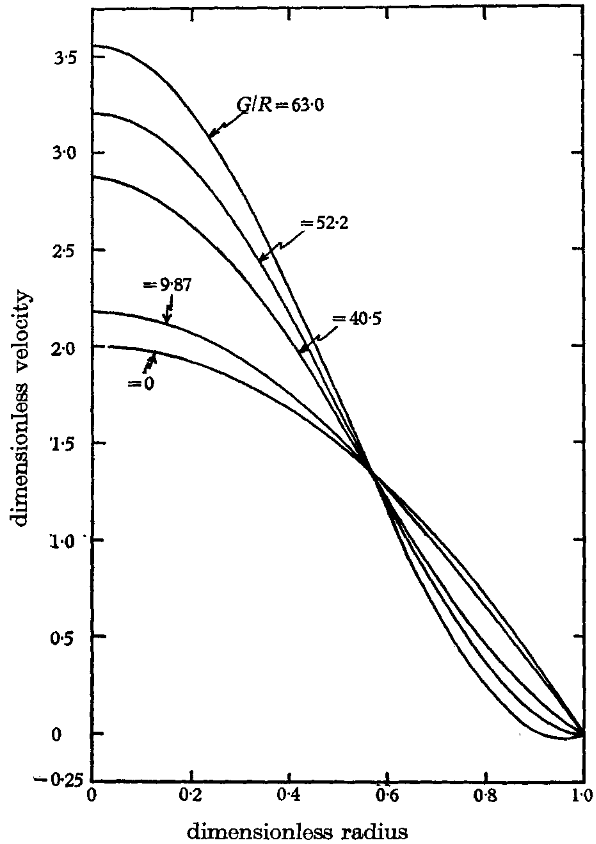

FIGURE 2. Velocity profiles for laminar downflow with constant-flux heating.

(heating in downflow) the velocity at the centre increases and the velocity at the wall decreases as the heat flux is increased. For $G/R = 9{\cdot}87$ there is a point of inflexion at the wall and at greater G/R the point of inflexion moves radially toward the centre of the pipe. At $G/R = 52{\cdot}2$, the velocity gradient at the wall becomes zero and a reversal of flow occurs at the wall for further increase in the ratio of heat flux to flow rate.

When heating or cooling with a constant-temperature wall no invariant distorted velocity profile is established in the pipe; rather, the profile continuously changes until, if the heat-transfer section is of sufficient length, the flow field reassumes a parabolic velocity distribution. Rosen & Hanratty (1961) used boundary-layer theory to obtain an approximate solution for the shape of the velocity profile as a function of axial location in the heat-transfer section.

At the location of maximum profile distortion the velocity and temperature fields may be described by the same equations as are used for fully developed flow with constant flux heating. Therefore, figures 1 and 2 also represent the velocity field that would be obtained for a constant-temperature wall at the location of maximum profile distortion. However, for the case of a constant-temperature wall it is convenient to use a parameter different from the group G/R defined in this paper to characterize the system.

Hanratty *et al.* (1958) and Scheele, Rosen & Hanratty (1960) described visual dye experiments which illustrate the steady and unsteady flow patterns obtained for heating or cooling water flowing in a vertical pipe. Their experiments indicated that changes in the flow field resulted primarily from natural convection effects rather than viscosity variation since the same results were obtained for heating in upflow as were obtained for cooling in downflow. Unsteady flows were noted at Reynolds numbers as low as 50 for temperature differences between the wall and the inlet fluid as low as 10° F.

For heating in upflow with a constant-temperature wall and with large enough temperature differences between the wall and the inlet fluid a region of reversed flow was obtained in the central portion of the pipe, and downstream this inverted flow became unsteady. The upstream region of reversed flow appeared quite stable and could be illustrated by flooding the flow field with dye and later purging the dye away. Dye remained in regions of low velocity and the paraboloid in figures 3 and 4, plate 1, appeared.* The tip of this paraboloid was a stagnation point in the flow field and a reversed flow existed inside the paraboloid. The location of the tip of this paraboloid in the pipe was used to check the analysis of Rosen & Hanratty (1961). Although transition to an unsteady flow always accompanied a reversed flow when natural convection was in the direction of forced flow, it was found that it is not necessary to have a reversal in order for the flow to become unstable. Scheele *et al.* (1960), on the basis of both constant-wall-temperature and constant-heat-flux experiments, suggested that instability first occurs when the profiles develop points of inflexion ($G/R = 32{\cdot}94$), but were unable to substantiate their hypothesis because the heat-transfer section they used was too short.

When natural convection is opposed to the direction of forced flow, the transition to an unsteady flow was preceded by an asymmetry in the flow field. The transition was associated with a reversal of flow, and therefore separation. Owing to the asymmetry, the flow separated initially only over part of the circumference of the pipe wall. At small Reynolds numbers reversed steady flows over a portion of the wall were obtained. These reversed flows were demonstrated quite vividly by purging the flow field after it was flooded with dye and also by watching the motion of a dye filament in the centre of the pipe. Asymmetries were also noted in the wall temperatures in all of the downflow heat-transfer experiments of Hallman (1961).

The mechanism of transition from a steady to an unsteady flow was found to depend on whether the natural convection was in the direction of or opposite

* Motion pictures of these experiments are available on request from Visual Aids Service, University of Illinois, 713 and 1/2 South Wright Street, Champaign, Illinois.

the direction of forced flow. When natural convection and the forced flow were in the same direction transition occurred through a gradual growth of small disturbances. The first instability was a slight sinuous motion of the dye filament noted at the outlet of the heat-transfer section. For increased heat input or decreased flow rate the sinuous fluctuations grew in amplitude and became distorted until eventually the dye filament broke up and mixed with the other fluid. The dependency of the transition upon the length of heat-transfer section indicates that a certain length of time is necessary for the development of flow disturbances to a condition such that they would be visible. Although a particular flow is unstable, transition need not be observed. Thus, the whole region of reversed flow shown in figures 3 and 4, plate 1, is unstable (unless the changes in the momentum of the flow profile act as a stabilizing influence); however, transition was observed only in the downstream portion of the reversed flow. When natural convection was opposed to the direction of forced flow, transition occurred suddenly. Transition was noted by intermittent bursts of highly disturbed flow followed by periods of undisturbed flow.

The experiments described above indicated that the transition to a disturbed flow is related primarily to the velocity profile distortion and, in the case of natural convection in the direction of forced flow, to the time available for the growth of small disturbances. However, there were two limitations to the work. In the first place the use of dye injection techniques limited the study to Reynolds numbers less than 350 and, secondly, the small length of heat-transfer section available (length to diameter ratio, L/D, equal to 114) for the growth of disturbances made it difficult to establish which profiles were unstable. Therefore, it was decided to carry out transition studies in much longer heat-transfer sections. A constant heat flux was used rather than a constant-temperature wall since for a given flow rate and heat flux an approximately fully developed flow is attained downstream in the heat-transfer section and the stability of this particular velocity profile can be studied. By performing both upflow and downflow experiments, natural convection with and opposed to the forced flow was studied. By use of a carefully designed entry and a long isothermal section prior to the heat-exchange section, inlet disturbances were eliminated and the transition resulted from a natural instability of the distorted profiles. Transition was detected by measuring the temperature fluctuations at the outlet of the heat-transfer section. The results of these experiments are reported in this paper. In addition to being of interest in the study of the effect of heat transfer on transition to a disturbed flow they should be of interest for the application of hydrodynamic stability theory since this research constitutes a study of the stability to infinitesimal disturbances of pipe flows having a variety of velocity profile shapes.

Experimental

The study was conducted in a 60 ft. vertical length of 0·787 in. inner diameter copper pipe. The downstream 50 ft. was wrapped with Chromel-A heating ribbon and surrounded with a 1 in. thickness of 85 % magnesia insulation to reduce radial heat loses. There were five independent 10 ft. heating sections, and it was thus possible to vary the length of the heat-transfer section from 10 to 50 ft. in 10 ft. increments. The maximum length of the heat-transfer section was 762 pipe diameters and the length of the calming section varied from 152 to 762 pipe diameters, depending on the length of the heat-transfer section used. For upflow runs partially deaerated room-temperature water contained in two 140 gallon tanks connected in parallel was forced upward by compressed air. For downflow runs the water was fed by gravity to the test section from a 250-gallon tank. At the inlet end of the pipe there was a tapered entry section with a flared inlet which extended into the feed tank. With this entry and with a minimum of disturbance in the feed tank it was possible to obtain isothermal laminar flow for R values in excess of 5000. Temperature fluctuations were detected with a 30-gauge copper-constantan thermocouple inserted into the centre of the pipe at the downstream end of the heat-transfer section. The d.c. portion of the signal was filtered out and the a.c. portion was amplified and recorded using a Minneapolis-Honeywell 'Viscicorder'. It was not possible to detect temperature fluctuations of less than about 0·1 °C because for signal fluctuations of 5 μV or less the noise level was comparable to the signal level. The thermocouple had a poor high-frequency response and could not respond accurately to fluctuations greater than about 5 c/s. Wall-temperature measurements were made with 30-gauge copper-constantan thermocouples embedded in the wall of the pipe. The inlet water temperature was determined from measurements of wall temperatures in the isothermal calming section, and the outlet average temperature was measured by a thermocouple placed downstream of a disturbance plate located at the end of the heat-transfer section. The actual heat input to the water was determined from an overall heat balance using measured inlet and outlet water temperatures. This heat input was always less than the power input to the heaters because of radial heat losses, which were small. The assumption that the heat flux was constant over the heated length of pipe in the experiments is valid if the radial heat losses were constant over the length, since the power input to each heating element was the same. This assumption was found to be a good one, since for a given power input per heating element and a constant flow rate no appreciable variation of heat input to the water per unit length was observed as the heated length of pipe was changed.

Experimentally, it is not possible to establish a completely invariant velocity profile in the downstream end of the heat-transfer section. Although the velocity profile distortion is due primarily to natural convection effects, there is a slight variation of G/R in the heat-transfer section due to variations of the kinematic viscosity and the coefficient of expansion of the fluid with temperature. The variation in the theoretical velocity profile over the last 20 ft. of heated pipe for conditions near transitions is shown in figure 5, for upflow heating. Transition was defined as the condition for which fluctuations in the fluid temperature at the outlet of the heat-transfer section were first observed. The transition values of G/R presented in this paper are the arithmetic average of the highest G/R for which no temperature fluctuations were detected and the lowest G/R for which there were fluctuations in the fluid temperature. The averaged values of G/R are within 5 % of the values from which the average was obtained.

Wall-temperature measurements for laminar flow conditions just prior to transition gave an indication of the extent of the region of fully developed temperature profiles in the pipe. A comparison of these measurements with wall temperatures calculated from theory gives an indication of the accuracy of the theory in predicting velocity profiles. Typical wall-temperature profiles for

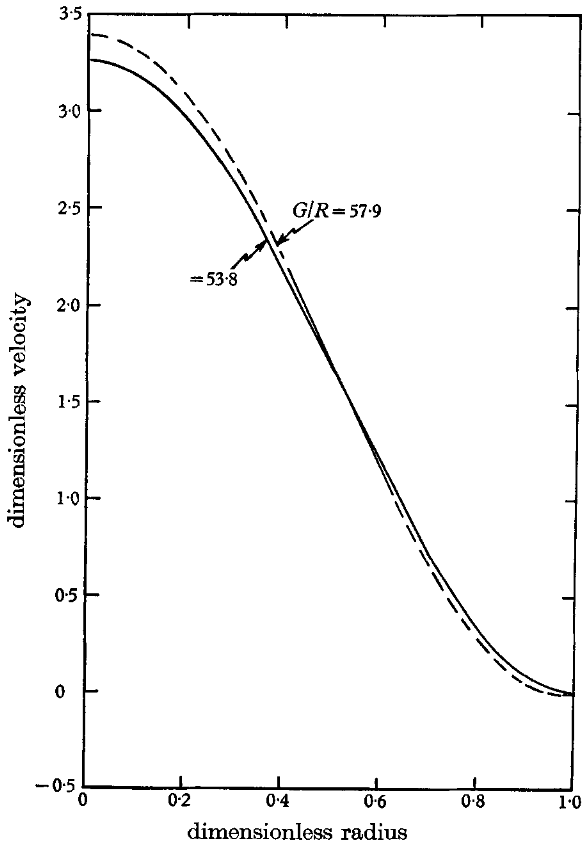

FIGURE 5. Variation in fully developed distorted laminar velocity profile for downflow with constant-flux heating for $L/D = 305$.

upflow at low and high flow rates are shown in figure 6 along with wall temperatures predicted by theory. The length of heated pipe necessary to obtain a fully developed temperature field is seen to increase with increasing flow rate. In the present study, temperature profiles were fully developed over a portion of the heat-transfer section for all upflow runs but not for all downflow runs. For upflow heating the agreement between experiment and theory over the entire Reynolds number range of 40–1800 investigated is sufficiently good to support the assumption that natural convection effects predominate in distorting the velocity profile and that theory does predict the flow-field distortion. For downflow the agreement between experimental and theoretical Nusselt numbers before transition is not so good as in upflow. At R less than about 600 the experimental values are lower, at R greater than 600 they are greater, though over the range $R = 300$ to 800 the disagreement between theory and experiment does not appear too large.

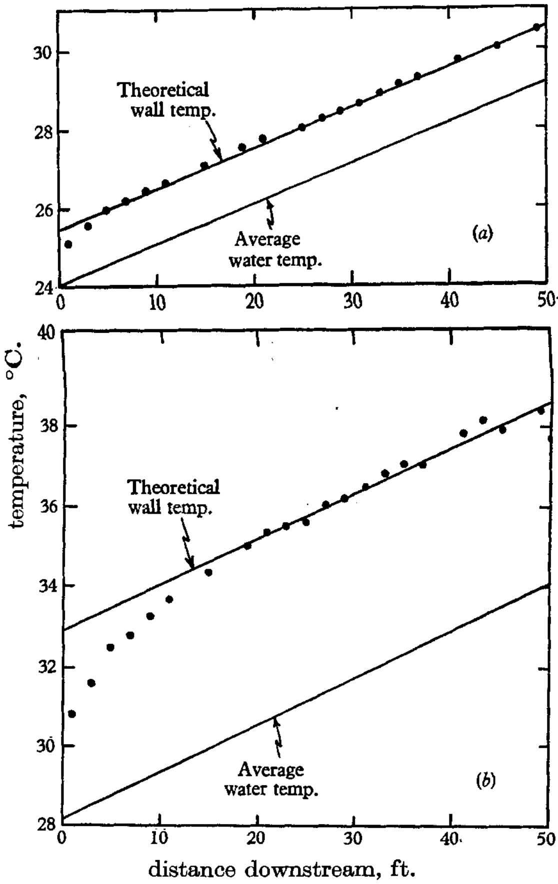

FIGURE 6. Wall-temperature behaviour for laminar upflow with constant-flux heating: (a) $R_{\text{inlet}} = 339$, $(G/R)_{\text{inlet}} = 30{\cdot}2$; $R_{\text{outlet}} = 380$, $(G/R)_{\text{outlet}} = 40{\cdot}3$; (b) $R_{\text{inlet}} = 1132$, $(G/R)_{\text{inlet}} = 44{\cdot}2$; $R_{\text{outlet}} = 1283$, $(G/R)_{\text{outlet}} = 58{\cdot}7$.

Results on transition

The results of this study have been tabulated in detail in a thesis by Scheele (1962).

Since for symmetrical laminar flow the shape of the velocity profile is a function only of G/R the transition conditions have been plotted as G/R versus R with L/D as a parameter in figures 7, 8, and 9. In figures 7 and 8, fluid properties have been evaluated at the outlet temperature and in figure 9 they have been evaluated at the inlet temperature. It can be seen that the length-to-diameter ratio of the heat-transfer section influences the data at large flow rates. The data for $L/D = 114$, obtained by dye techniques, were reported in the paper by Scheele *et al.* (1960) and are tabulated in a thesis by Scheele (1959). For upflow heating, if fluid properties are evaluated at the outlet temperature, the results asymptotically approach a G/R lower limit of $42{\cdot}5$ as the flow rate decreases.

If fluid properties are evaluated at the inlet temperature G/R approaches a limit of about 33. In contrast to the transition data for upflow heating the downflow transition values of G/R approach no asymptotic minimum value at low flow rates. Instead, as the flow rate increases, G/R at transition decreases to

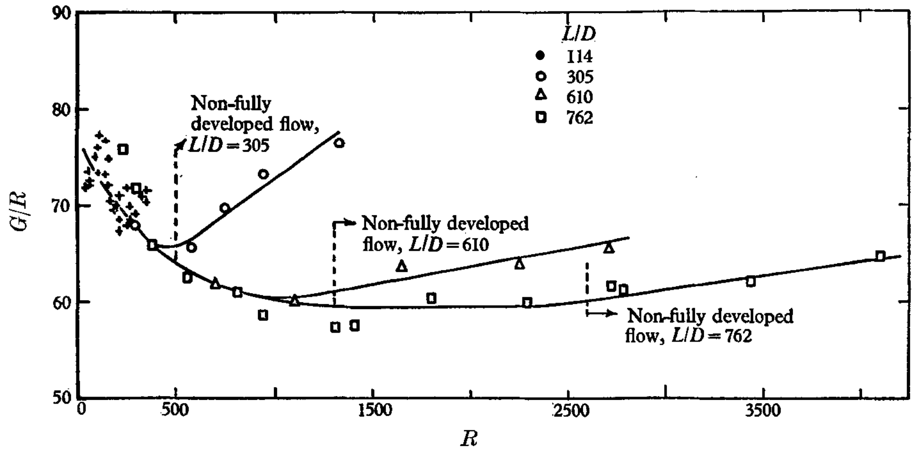

FIGURE 7. Transition for downflow with constant-flux heating; fluid properties evaluated at outlet temperature.

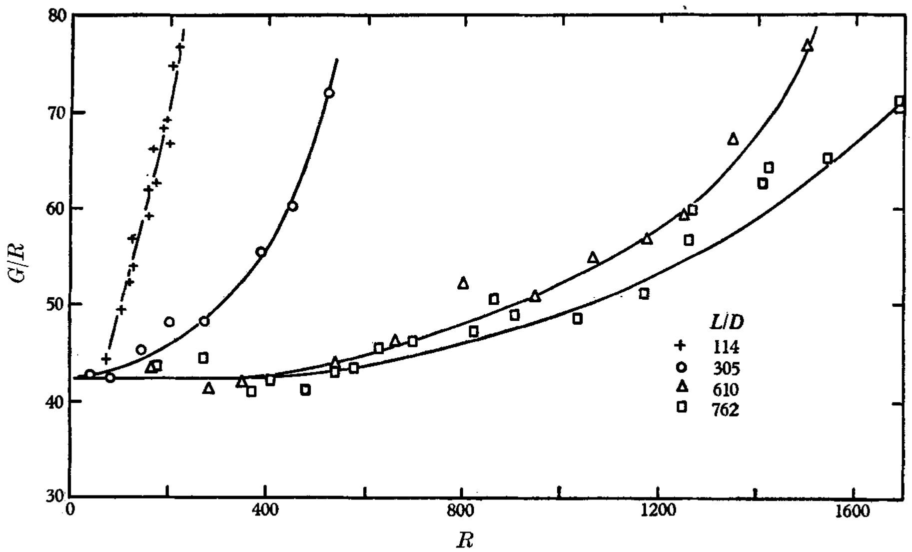

FIGURE 8. Transition for upflow with constant-flux heating; fluid properties evaluated at outlet temperature.

some minimum value dependent on the heated length of pipe and then gradually increases as the flow rate is increased further. In the downflow experiments at high flow rates the wall-temperature measurements indicated that the length of the heat-transfer section was not sufficient to obtain a fully developed temperature profile. The values of R above which fully developed temperature profiles did not exist are indicated in figure 7.

The character of the temperature fluctuations show the transition processes for upflow heating and downflow heating to be quite different. For upflow heating at a value of R of about 200 and G/R of about 45 the temperature fluctuations are low-frequency small-scale oscillations. As G/R increases the fluctuations increase in amplitude but, even for $G/R = 750$, no turbulent-like fluctuations

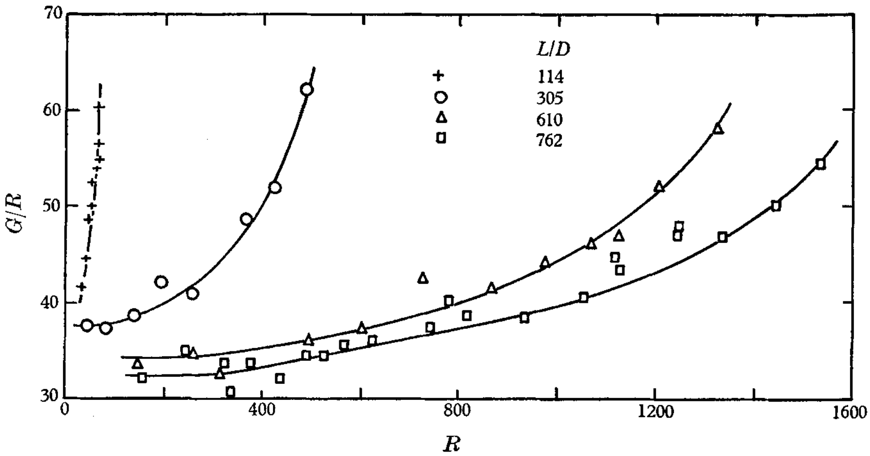

FIGURE 9. Transition for upflow with constant-flux heating; fluid properties evaluated at inlet temperature.

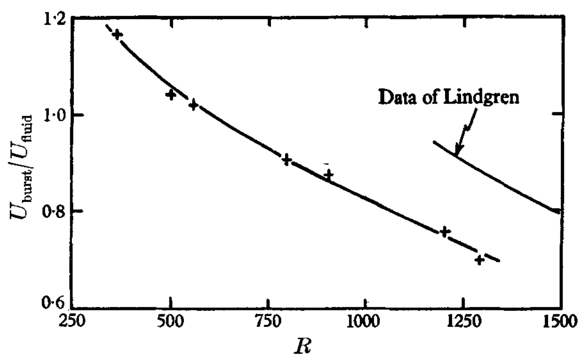

FIGURE 10. Relative velocity of rear of turbulent burst with respect to average fluid velocity for downflow with constant-flux heating.

are observed. At a value of R of about 540 the initially regular fluctuations distort and grow in amplitude until fluctuations characteristic of turbulence appear for values of G/R of about 225. However, even at quite large values of G/R the fluctuations have a large-scale periodicity associated with them. At a value of R of about 1650 the initially regular oscillations are rapidly distorted with increase in G/R and form discrete bursts. Further increase in the ratio of heat input to flow rate produces fluctuations characteristic of turbulence.

For downflow heating at all Reynolds numbers the first deviations from steady laminar flow are intermittent bursts of disturbed flow. As G/R is increased, the intermittent laminar and disturbed flow pattern persists, but the proportion of the time for which the flow is laminar decreases. Eventually the flow loses its intermittent character and appears to become fully turbulent. A sharp decrease in the wall temperature was noted when the tail end of a turbulent burst passed a given location. When the flow becomes fully turbulent the wall temperature fluctuations are of smaller magnitude. The sharp drop in the wall temperature accompanying the passage of the tail of a turbulent burst makes it possible to measure the rear velocity of these large-scale bursts by recording wall temperatures at two axial locations. Figure 10 shows the relative velocity of the tail of the turbulent bursts with respect to the average fluid velocity. Data on burst velocities obtained by Lindgren (1961) for isothermal flow of a 0·062% White Hector bentonite sol are also shown in figure 10. Drawings of the recorded temperature fluctuations are shown in a thesis by Scheele (1962).

Interpretation of transition results

Transitions to a disturbed flow for upflow heating and for downflow heating occur over a relatively small range of G/R. Since the shape of the velocity profile depends primarily on G/R it appears that the stability of a particular flow depends primarily on the shape of the velocity profile and only secondarily on the value of R, if at all. As indicated in figure 1, for upflow heating the velocity profile develops a dimple at the pipe centre and possesses two points of inflexion for $G/R = 32{\cdot}94$. The transition measurements indicate that the flow field first becomes unstable when the velocity profile develops these inflexion points. This is evidenced by the fact that at low R the transition G/R for the longest heat-transfer section is approximately constant and equals about 33 when the fluid properties are evaluated at the inlet temperature and about 42·5 when the fluid properties are evaluated at the outlet temperature. If, as was suggested by Scheele *et al.* (1960), the transition is not instantaneous and instabilities require a considerable time before they can grow to an amplitude that can be detected, fluctuations detected at the outlet of the heat-transfer section were probably initiated far upstream. Therefore, at low values of R, it would be expected that if G/R is evaluated at the inlet conditions it would be more representative of the profile at which the instability was initiated than if it were evaluated at the outlet conditions. As the Reynolds number is increased or as L/D is decreased, the length of pipe in which the flow field and the temperature field are fully developed becomes shorter and greater values of G/R are needed in order for transition to be noted. The larger values of G/R will cause the velocity profile to become unstable early in the heat-transfer section before a fully developed field is obtained, and, perhaps, will cause a larger rate of growth of disturbances because of the greater distortion of the velocity profile. Thus, the effect of R and L/D on the transition measurements for upflow heating can be explained in terms of a transition mechanism involving the slow growth of infinitesimal disturbances.

The measurements of Scheele *et al.* (1960), as well as the measurements reported in this paper, indicate that for heating in downflow the transition is rather sudden once the velocity profile becomes unstable. Therefore, it seems appropriate to evaluate G/R at the conditions at which transition is first noted. The G/R at transition at all values of R is greater than 52·2, the value calculated for a symmetrical flow field with zero velocity gradient at the wall. The effect of L/D on transition and the slight increase in the transition value of G/R at large Reynolds numbers can be explained by the fact that the heat transfer section was not long enough to obtain a fully developed flow. These transition data, in addition to temperature fluctuation data and the experiments reported by Scheele *et al.* (1960), indicate that transition for downflow heating over the entire Reynolds number range is initiated by flow separation at the wall. Scheele *et al.* observed that at low Reynolds numbers stable asymmetric reversed flows at the wall were possible whereas at large Reynolds numbers the asymmetric reversed flows lead quite rapidly to a transition to an unsteady motion. The increase in the transition value of G/R at small Reynolds numbers noted in figure 7 can therefore be explained if there is at small R a range of G/R for which steady reversed flows at the wall can exist. At large Reynolds numbers the transition G/R appears to approach a constant value of about 59, which is close to the value of 52·2 predicted for separation at the wall. Although the difference in these two values of G/R as well as in measured and predicted heat-transfer coefficients prior to transition can be explained by either viscosity variation or asymmetry in the flow field, it is felt that the differences between theory and experiment for downflow heat transfer at large R are due primarily to asymmetries in the flow field. This is based on the fact that previous investigations have indicated asymmetries in the flow field when natural convection opposed forced convection. The fact that measured heat-transfer coefficients at low R differed from those obtained at large R, in that they were smaller than predicted, might be explained by the fact that they were measured under conditions such that a steady reversed flow existed over a large portion of the heat-transfer section. Since the presence of an inflexion point in the distorted velocity profiles leads to transition for upflow heating, it would be expected for downflow heating that a transition would occur in the neighbourhood of $G/R = 9{\cdot}87$. That no transition to an unsteady flow was observed could be explained if inflexion points in the centre of the pipe are more unstable than inflexion points near the wall. However, it is more likely that points of inflexion never existed in the downflow experiments because at $G/R = 9{\cdot}87$ there is a transition from a symmetrical steady flow to an asymmetrical steady flow.

Nature of the unsteady flow

For heating in upflow the measurements of temperature fluctuations in the centre of the pipe did not show a random structure characteristic of turbulent flow unless the Reynolds number was close to the value of 1050 needed to sustain a turbulent field for isothermal flow. However, this observation might just reflect the insensitivity of the measuring instrument.

For heating in downflow apparently random temperature fluctuations were noted at large enough G/R for Reynolds numbers much smaller than are needed to sustain turbulence for isothermal flow. The intermittent character of the temperature fluctuations noted after transition to an unsteady flow could be interpreted as an alternating separation and reattachment process as suggested by Guerreri & Hanna (1952) in conjunction with their heat-transfer experiments in a rectangular channel. Perhaps the role of the temperature field in sustaining turbulence at low Reynolds numbers can be explained in terms of a separation at the wall which is occurring at very high frequency.

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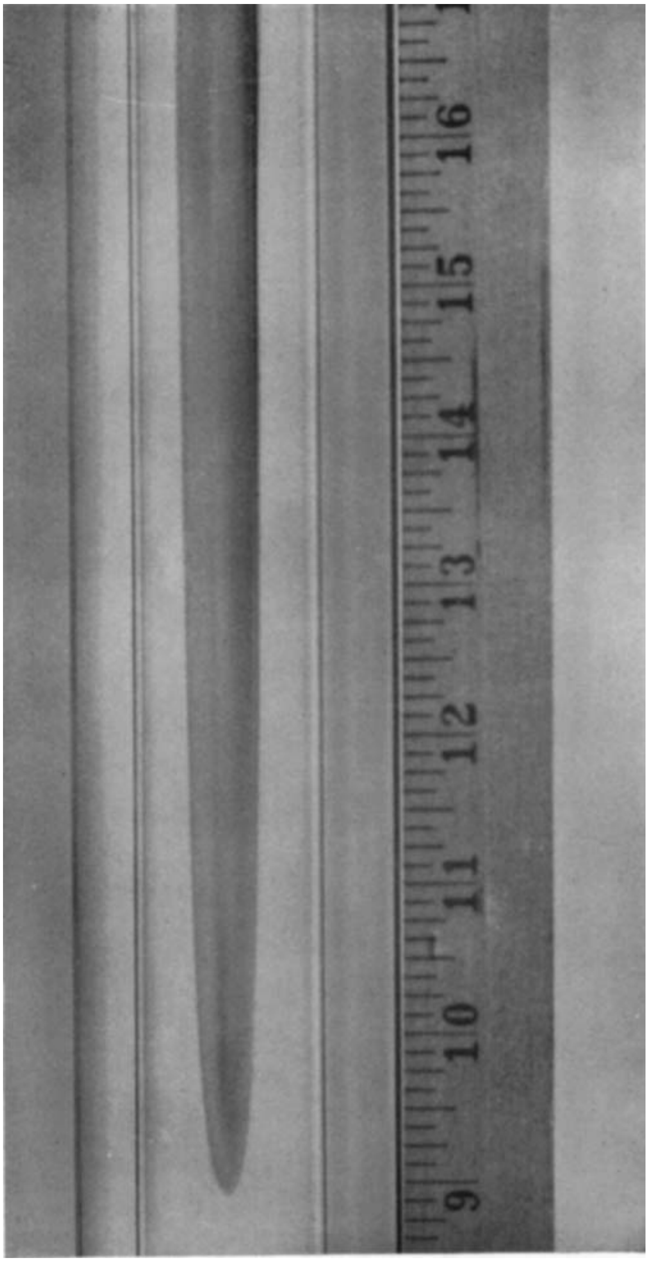

FIGURE 3. Detail of upstream flow pattern, apex of paraboloid, at $R = 125$.

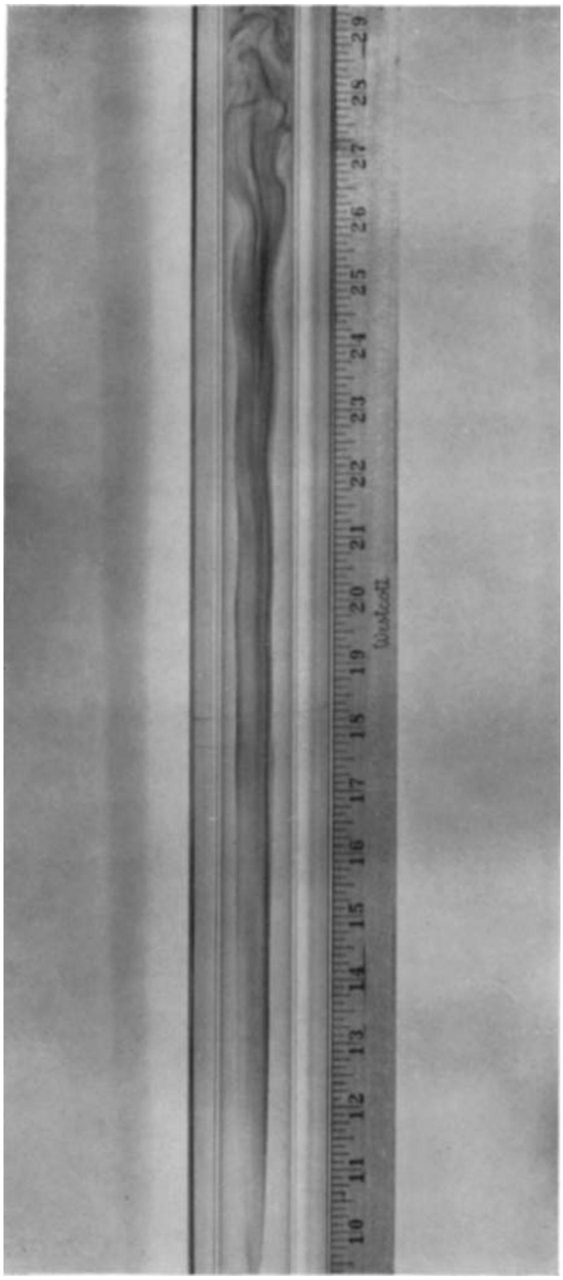

FIGURE 4. Dye flow pattern obtained by heating water in upflow at $R = 125$.